

MATHEMATICS 5 PERIODS PART B

DATE: 6th June 2016, morning

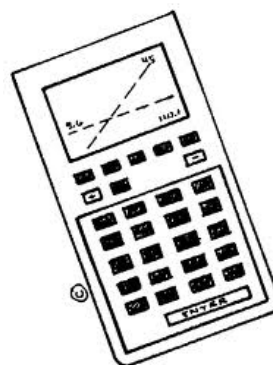
DURATION OF THE EXAMINATION:

3 hours (180 minutes)

AUTHORIZED MATERIAL:

Examination with technological tool :
Calculator TI-Nspire in “Press-to-test” mode

Pencil for the graphs



SPECIFIC INSTRUCTIONS:

- Use a different examination sheet for each question.
- Answers must be supported by explanations.
- They must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.
- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.
- When the answer provided is not the correct one, still some marks can be awarded if it is shown that an appropriate method and/or a correct approach has been used.

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PART B		
QUESTION B1 ANALYSIS	Page 1/1	Marks
<p>Use the calculator in b), d) and e).</p> <p>Consider the family of functions f_t, with $t \geq 0$, given by</p> $f_t(x) = \frac{3}{x^2 - 3x + t}.$		
a) Determine the domain of definition of each of the functions f_0 and f_3 .		2 marks
b) Sketch the graph of f_0 in the interval $-2 \leq x \leq 5$. Determine an equation of each of the asymptotes to the graph of f_0 .		4 marks
c) Calculate the exact coordinates of the extremum of f_0 .		3 marks
d) Calculate the area of the region bounded by the graph of f_3 and the x-axis.		3 marks
e) Consider the region bounded by the line $x = c$, where $c > 0$, the coordinate axes and the graph of f_3 . Determine the value of c such that the area of this region equals 1.2.		3 marks
f) Determine the values of t for which the graph of f_t has no asymptote parallel to the y-axis.		3 marks
g) Find the value of t for which the graph of f_t has no extreme points.		2 marks

PART B		
QUESTION B2 GEOMETRY	Page 1/1	Marks
<p>In a 3 dimensional space consider</p> <p>the sphere S with centre $T(3,3,3)$ and radius $3\sqrt{3}$,</p> <p>the points $O(0,0,0)$, $A(6,0,0)$, $B(0,6,0)$ and $C(0,0,6)$,</p> <p>the plane $\alpha: 2x - 4y - z - 6 = 0$ and</p> <p>the vector $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.</p> <p>a) Determine an equation of the sphere S.</p> <p>b) Verify that there is no point of the tetrahedron $OABC$ outside the sphere S.</p> <p>c) Find an equation of the plane tangent to the sphere S at the point A.</p> <p>d) Determine the coordinates of the point P, the orthogonal projection of the point O on the plane π containing the points A, B and C.</p> <p>e) Use your calculator to find the acute angle in degrees between the planes α and the xz-plane.</p> <p>f) There is a line, with direction vector \vec{v}, intersecting the line AB and the line OC. Determine a system of parametric equations of this line.</p> <p>g) Show that the plane α intersects the line segment AC.</p>		<p>2 marks</p> <p>3 marks</p> <p>3 marks</p> <p>4 marks</p> <p>3 marks</p> <p>3 marks</p> <p>2 marks</p>

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QUESTION B3 PROBABILITY	Page 1/1	Marks
<p>Use your calculator for all calculations in this question.</p> <p>The weights of newly born babies in Memorial Hospital are normally distributed with a mean of 3500 g and a standard deviation of 300 g.</p> <p>a) Calculate the probability that a newly born baby in Memorial Hospital will have a weight over 3800 g.</p> <p>b) Calculate the probability that a newly born baby in Memorial Hospital will have a weight between 3000 g and 3600 g.</p> <p>The weights of newly born babies in Elisabeth's Hospital are normally distributed with mean μ and standard deviation σ. In Elisabeth's Hospital 2.28 % of the newly born babies have a weight of over 3800 g and 11.5 % have a weight of less than 3000 g.</p> <p>c) Calculate the mean μ and the standard deviation σ.</p> <p>From now on for Elisabeth's Hospital use $\mu = 3300$ g and $\sigma = 250$ g . Any newly born baby with a weight that is within two standard deviations from the mean is said to be in the "normal-weight-range".</p> <p>d) Verify, by calculation, that 95 % of newly born babies in Elisabeth's Hospital are in the "normal-weight-range".</p> <p>e) Given that a newly born baby in Elisabeth's Hospital is in the "normal-weight-range", calculate the probability that its weight is over 3000 g.</p> <p>f) Calculate the probability that out of 220 newly born babies in Elisabeth's Hospital more than 210 will be in the "normal-weight-range".</p> <p>Incoming phone calls are received at the hospital at an average of 4 per minute. The number of incoming phone calls per minute follows a Poisson distribution.</p> <p>g) Calculate the probability that in a one minute interval there will be 5 or more incoming phone calls.</p> <p>h) Calculate the probability that in a two minute interval there will be less than 7 incoming phone calls.</p>		<p>2 marks</p> <p>2 marks</p> <p>4 marks</p> <p>2 marks</p> <p>3 marks</p> <p>3 marks</p> <p>2 marks</p> <p>2 marks</p>

PART B		
QUESTION B4 SEQUENCES	Page 1/1	Marks
<p>The sequences (u_n) and (v_n) are defined by:</p> $\begin{cases} u_1 = 5 \\ u_{n+1} = \frac{3}{5}u_n - 2 \end{cases} \quad \text{and} \quad v_n = u_n + 5, \quad n \geq 1.$ <p>a) Show that (v_n) is a geometric sequence with common ratio $\frac{3}{5}$.</p> <p>A further sequence (A_n) is defined by $A_n = v_1 + v_2 + \dots + v_n$.</p> <p>b) Express A_n as a function of n, then calculate $\lim_{n \rightarrow +\infty} A_n$.</p>		<p>2 marks</p> <p>3 marks</p>

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PART B		
QUESTION B5 COMPLEX NUMBERS	Page 1/1	Marks
<p>Consider the complex numbers $z = 1 + i\sqrt{3}$ and $w = \sqrt{3} + i$.</p> <p>Calculate $z \cdot w$, and determine the positive integers n for which $(z \cdot w)^n$ is a negative real number.</p>		5 marks